

*Numbers, their Minders and Keepers: good, bad,
mad, and sometimes sad.*

U3A Winter School 2018

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Mathematics and Mathematicians

Charles Darwin: A mathematician is a blind man in a dark room looking for a black cat which isn't there.

Leonardo da Vinci: No human investigation can be called real science if it cannot be demonstrated mathematically.

Goethe: Mathematicians are a species of Frenchmen: if you say something to them they translate it into their own language and presto! It is something entirely different.

A.E. Whitehead : The pursuit of mathematics is a diverse madness of the human spirit, a refuge from the goading urgency of contingent happiness.

Thomas Mann: I tell them that if they will occupy themselves with the study of mathematics they will find in it the best remedy against the lusts of the flesh.

Anonymous student:

Dear Mathematics,

I am sick and tired of trying to find your 'x'. Just accept the fact that she is gone and move on.

Mystique of Mathematics - Proof by Intimidation - Euler's 'proof' of God

Leonhard Euler (1707 – 1783): Swiss mathematician and son of a Calvinist minister.

Denis Diderot (1713 – 1784): French philosopher and atheist.

Venue: Court of Catherine the Great of Russia.

Concerned about Diderot's influence on the court she asks Euler for help.

Euler: Sir,

$$\frac{a+b^n}{n} = x$$

hence God exists. Refute!

Precocious talent

Carl Friedrich Gauss (1777 - 1855): German mathematician.

School teacher's nightmare – add the integers from 1 to 100.

G H. Hardy (English mathematician)

I remember once going to see him [Ramanujan] when he was lying ill at Putney. I had ridden in taxi-cab No. 1729, and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavourable omen. "No, no Hardy" he replied, "it is a very interesting number; it is the smallest number expressible as a sum of two cubes in two different ways." $[1^3 + 12^3 \text{ AND } 10^3 + 9^3]$

The fascination of integers

Leopold Kronecker (1823 – 1891): God made the integers, all the rest is the work of man.

Positive integers (also called *natural numbers* or *counting numbers*):

1, 2, 3, 4, 5, 6, 7...99, 100, ... 12345, ... 13789257... infinitely many

Prime number: positive integer whose only *divisors* are itself and 1 e.g. 2, 3, 5, 13, 37 etc

The largest known prime as of now is $2^{77,232,917} - 1$ (or M77232917 for short). Discovered on December 26, 2017 it contains 23,249,425 digits – about 6643 A4 pages of print.

(An A4 page with standard margins contains 3500 digits - 50 lines of 70 digits per line using 12 pt type.) Contender for the [World's Most Boring Book](#) if printed out.

Composite numbers: the rest e.g. $6 = 3 \times 2 \times 1$; $12 = 4 \times 3 \times 1 = (2^2 \times 3 \times 1)$

Fundamental Theorem of Arithmetic: Every positive integer can be written uniquely as the product of powers of prime numbers.

e.g. $436, 640, 568 = 8 \times 3 \times 49 \times 371$ $293 = 2^3 \times 3^1 \times 7^2 \times 13^5$

Encrypting information – security on the Internet (RSA system)

RSA stands for Ron Rivest, Adi Shamir and Leonard Adleman, who first published the method in 1977

<https://www.my.commbank.com.au> (**https** indicates site is secure)

Is based on the product of two large *prime numbers* plus manipulations – easy to find their product, but virtually impossible to work backwards from a starting number - which is needed for code breaking.

e.g. $8191 \times 524287 = 4,294,434,817$ ($L \rightarrow R$ easy; $L \leftarrow R$ hard)

A message encrypted using two 16-digit numbers gives

340,282,366,920,938,463,463,374,607,431,768,211,456 possibilities.

Using present day computing power the time to crack such a code by calculation is estimated to be 1.1×10^{16} millenia. (11,000,000,000,000,000,000 years)

Recall: Largest known prime number has 23,249,425 digits *not* 16!

How barcodes work

In 1974 the first retail product (a packet of chewing gum) was sold using a barcode reader at a supermarket in Ohio. Australia uses the *European Article Numbering Code* containing 13 digits (EAN-13),



An example of an EAN-13 barcode

Examples: country code (2) + manufacturer (5) + product (5) + check digit (1)

Australia Farmland tomato sauce: 93 00601 12804 **4**

Australia Farmland iodised salt : 93 00601 18014 **1**

First 12 digits labelled from the left from 0 (zeroth digit) to 11: 13th is check digit

$3x(1st + 3rd + \dots + 11th \text{ digit}) + 1x(0th + 2nd + 4th + \dots + 10th \text{ digit}) + \text{check digit}$ is a multiple of 10. (Here 3, and 1 are referred to as weights)

Tomato sauce: $3x(3+0+0+1+8+4) + 1x(9+0+6+1+2+0) + 4$ (adds to 70)

Iodised salt: $3x(3+0+0+1+0+4) + 1x(9+0+6+1+8+1) + 1$ (adds to 50)

How Barcodes Work (error detection)

If a label is distorted so that no beep sounds, a checkout attendant then enters the barcode by hand - and this procedure is also subject to error.

If a digit 'a' is mistyped as 'b' (for example 8 is typed as 6) the sum will change by $1(b-a)$ if the weight is 1, or by $3(b-a)$ if the weight is 3.

Errors will go undetected if these amounts are a multiple of 10: 10,20,30...

Now $1(b-a) = 10, 20..$, cannot be satisfied for unequal values of a and b that range between 0 and 9.

Also $3(b-a) = 10, 20$ cannot be met with whole number values for a and b

While $3(b-a) = 30$ is the same as $(b-a) = 10$ as in the above.

So so the method gives a 100% detection rate.

What if the weights were 1 and 2 instead of 1 and 3?

Then we would have $2(b-a) = 10$ instead of $3(b-a) = 10$.

$(b-a) = 5$ would be satisfied if 8 was mistyped as 3, or 2 was mistyped as 7 etc

Not all errors are detected.

1 and 3 are the simplest weights that guarantee 100% detection.

Before Enigma and the Imitation Game - Cracking the Code



Codes have fascinated humans for thousands of years. Elgar even wrote a secret code into his composition 'The Enigma Variations' just for amusement. Codes have been used in all eras and in all cultures.

A method of creating secret codes that was used in ancient times is known as the *Caesar Shift* made famous by Julius Caesar who used it to encrypt letters to his generals. To create the codes Caesar shifted every letter three places to the left in the alphabet. For example,

FRIENDS ROMANS COUNTRYMEN LEND ME YOUR EARS would become **COFBKAP OLJXKP ZLRKQOVJBK IBKA JB VLRO BXOP**.

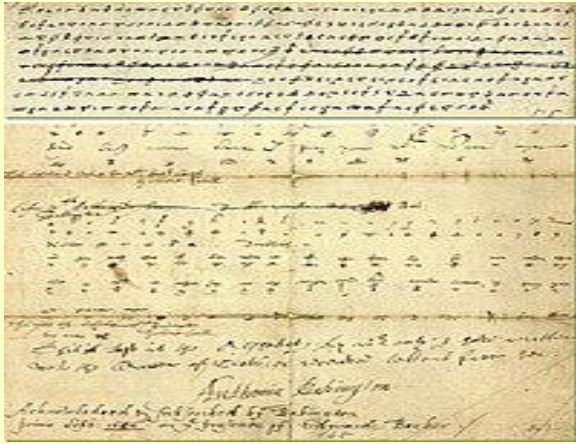


A Royal Plot

In 1586, Anthony Babington, communicating secretly with Mary Queen of Scots through coded letters, hatched a plot to assassinate Queen Elizabeth 1. However unbeknown to Babington and Mary the messenger was a double agent being paid by Elizabeth's principal secretary. The letters were passed on to a master code breaker who decoded the letters and Mary's fate was sealed. On February 8th, 1587 she was beheaded in the Great Hall of Fotheringay.

Babington used a substitution cipher in which symbols replaced the letters of the alphabet, with some meaningless "red herring" symbols also included. The code breaker Thomas Phellippes used frequency analysis to crack the code. This technique, featuring an *early use of statistics*, compared the relative frequencies with which letters occur in the English language with the relative frequencies that symbols occurred in the code.

a	b	c	d	e	f	g	h	i	k	l	m	n	o	p	q	r	s	t	u	x	y	z
o	†	λ	#	α	□	θ	∞	∩	δ	κ	∥	ϕ	∇	∫	∩	f	Δ	ε	c	7	8	9
Nulles ff. — . — d.												Dowbleth σ										
and for with that if but where as of the from by																						
2	3	4	+	4	3	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩
so not when there this in wich is what say me my wyrt																						
∩	x	+	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	
send lfe receive bearer I pray you Mte your name myne																						
∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	∩	



Code used by Mary

Extract from a letter by Mary to Babington. At the bottom are additions by Elizabeth's agent purporting to be Mary asking for the names of her loyal plotters.



Interesting Number Patterns

$$1 \times 8 + 1 = 9$$

$$12 \times 8 + 2 = 98$$

$$123 \times 8 + 3 = 987$$

$$1234 \times 8 + 4 = 9876$$

$$12345 \times 8 + 5 = 98765$$

$$123456 \times 8 + 6 = 987654$$

$$1234567 \times 8 + 7 = 9876543$$

$$12345678 \times 8 + 8 = 98765432$$

$$123456789 \times 8 + 9 = 987654321$$

Number eccentricities: Perfect, abundant, deficient, and happy numbers

Perfect number = sum of its *proper* divisors (*proper* divisors means that we leave out the number itself)

$6 = 1 + 2 + 3$; $28 = 1 + 2 + 4 + 7 + 14$. Next biggest are 496, and 8128

For 12: $1 + 2 + 3 + 4 + 6 = 16$ (greater than 12 so 12 is *abundant*)

For 14: $1 + 2 + 7 = 10$ (less than 14 so 14 is *deficient*)

Perfect numbers like perfect men are very rare – Rene Descartes

Amicable (friendly) numbers

Each is the sum of the proper divisors of the other: Show that (220 and 284) and (1184 and 1210) are pairs of amicable numbers.

Happy numbers

Take a positive integer N:

Replace N by the sum of the squares of its digits;

Repeat until the answer is 1 (where it stays - *happy*) or loops endlessly without reaching 1 (*sad*).

$13 \rightarrow 1^2 + 3^2 = 1 + 9 = 10 \rightarrow 1^2 + 0^2 = 1 + 0 = 1$ (*happy*)

$4 \rightarrow 16 \rightarrow 1 + 36 = 37 \rightarrow 9 + 49 = 58 \rightarrow 25 + 64 = 89 \rightarrow 64 + 81 = 145 \rightarrow 1 + 16 + 25 = 42 \rightarrow 16 + 4 = 20 = 4 + 0 = 4$ (*sad*).

Happy numbers (up to 100)

1,7,10,13,19,23,28,31,32,44,49,68,70,79,82,86,91,94,97,100

So 28 is perfectly happy

How many numbers?

Infinity (∞) is not a number as such.

The size (cardinality) of the set of integers is said to be aleph-null \aleph_0 the symbol being the first letter of the Hebrew alphabet.

Any set of numbers that can be matched 1-1 with the infinite sequence 1,2,3,4... has as many members as the positive integers and also has size \aleph_0

1 2 3 4 5 ...
1 2 3 4 5 ...

Double the numbers in the bottom line:

1 2 3 4 5 ...
2 4 6 8 10 ...

**There is still a 1-1 matching between the numbers in the two rows
– so there are as many even positive integers as positive integers
altogether.**

(The finite mind struggles with this idea)

Positive rational numbers (fractions)

Formed by ratios of positive integers.

Every rational number can be written as a decimal that *either* terminates *or repeats indefinitely*:

e.g. $\frac{1}{4} = 0.25$ (terminating decimal)

$\frac{1}{9}$ on a calculator gives 0.11111111 (as an approximation limited by display window)

But $\frac{1}{9} = 0.1111\dots$ exactly (unending decimal - not approximation).

$$N = 0.1111\dots$$

$$10N = 1.111\dots$$

Subtract: $9N = 1$ so $N = \frac{1}{9}$

$$\begin{aligned} 0.1111\dots &= 0.1 + 0.01 + 0.001 + 0.0001 + \dots \text{ for ever} \\ &= \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \dots \text{ for ever} \end{aligned}$$

A practical interpretation: A first step of $\frac{1}{10}$ metre is followed by succeeding steps each being one tenth the size of the previous one. If we go on for ever how far do we go? ($\frac{1}{9}$ metre)

An infinite number of steps but a finite distance!

But $1+2+3+4+\dots$ and $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}+\dots$ for example **DO** become infinitely large (diverge)

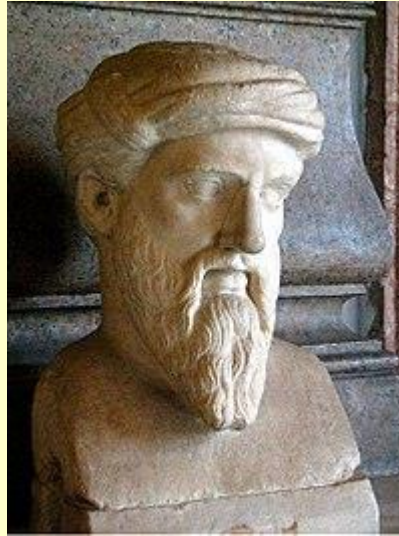
While $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} +\dots = 2$ (exactly) (converges like 0.1111...)

What is $N = 0.123123123\dots$ as an exact fraction?

$$1000N = 123.123123123\dots$$

Subtract: $999N = 123$ so $N = \frac{123}{999} = \frac{41}{333}$

The Pythagoreans



Pythagoras (about 570 – 495 BC)

Modest man: “There are men, gods, and men like Pythagoras”

Founded the Pythagorean society (school) – mystic rather than scientific.

Influence continued for several hundred years.

Basic philosophy

"Everything is created out of the whole numbers. From their ratios, differences and sums everything is made. The spheres are arranged by their immutable laws, rotating in eternal harmony. In the same way we can attain perfect harmony with the cosmos by opening our minds to the truth of numbers."

School motto – *all is number*

One – number of reason

Two – first even (female) number: number of opinion

Three - first odd (male number): number of harmony

Four – number of justice

.

Ten – number of the universe

Pythagorean society

- Egalitarian (men and women equal)
- Property communal (including mathematical discoveries)

Some school rules

- Abstain from beans
- Do not pick up what has fallen
- Don't poke a fire with iron

Mathematical knowledge and scandal

Arithmetic: prime numbers, odd and even numbers, composite numbers, perfect abundant and deficient numbers, amicable numbers, figurate numbers

Geometry: Various properties and theorems to do with triangles, parallel lines, polygons, circles, spheres, polyhedra.

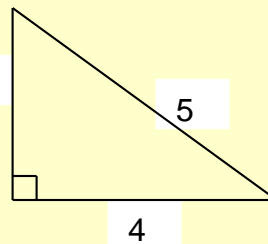
Shock Horror- Hipparsus

By the Pythagorean theorem:

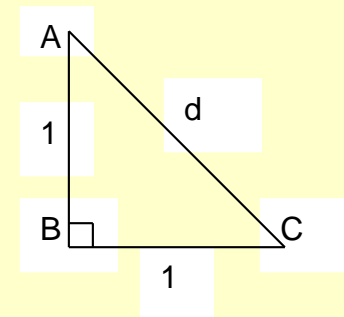
$$d^2 = 1^2 + 1^2$$

$$d^2 = 2$$

$$d = \sqrt{2}$$



$$5^2 = 3^2 + 4^2 \text{ (OK)}$$



Hipparsus showed that $\sqrt{2}$ could *not* be expressed using integers (a/b)

What to do about Hipparsus?

Posthumous pardon! “He is unworthy of the name of man who is ignorant of the fact that the diagonal of a square is incommensurable with its side.” Plato (429-347 BC).

International Standard Paper Sizes (A - series)

Aspect ratio (length/width) is preserved from size to size when each is cut in half length-wise.

$$L/W = W/(L/2)$$

$$L/W = 2W/L$$

$$R = 2/R \quad (\text{where } R = L/W)$$

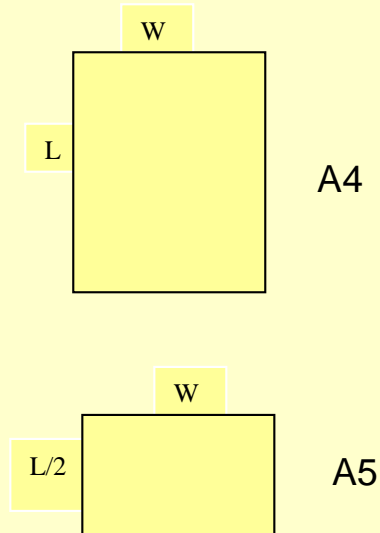
$$R \times R = R \times 2/R = 2$$

$$R^2 = 2$$

$$R = \sqrt{2}$$

$$R \approx 1.414$$

Measure dimensions and check for A3, A4, A5



Note: A0 (841mm x 1189 mm) has area of 1 square metre.

So the Pythagorean's nemesis permeates our everyday activity.

Another example: The ratio the of the distances of two consecutive frets to the bridge of a stringed instrument is $\sqrt[12]{2}$ or approximately 1.059463.

Number intrigues – growth patterns

Fold a newspaper page in half: then again, then again, then again....

If we keep doing this how high will the pile be after: 20 folds? 40 folds? 60 folds?

Number of folds (n)	Number of page thicknesses (P)	Pattern for P in terms of n
0	1	2^0
1	2	2^1
2	4	2^2
3	8	2^3
20		
40		
60	60	2^{60}

Estimate the thickness of a single page: for example a ream of 500 pages of copying paper is about 5.5 cm thick. (newspaper page = 0.5 x A4 copy page)

Estimate the total heights (H) for:

$n = 20$: $H = \dots$; $n = 40$: $H = \dots$; $n = 60$: $H = \dots$

Some geographical distances

Mt Everest: 8848 m ; Mariana's trench (deepest ocean) 10,916 m; Sydney-Beijing by air: 8923 km Earth's diameter: 12742 km; Australia's mainline coastline: 35,877 km! Earth to moon: 384,403 km; Earth to sun: 149,597,890 km

How were the guesses? (60 folds - over 420 times distance from earth to sun)

Tower of Wanaqa! Hanoi! Brahma!



Goal: To move the entire pile of discs from one of the three pegs to another peg in the minimum number of moves such that:

- The discs are moved one at a time
- A larger disc cannot be placed on a smaller one

Note: The full tower contains 64 discs

Tower of Wanaka! Hanoi! Brahma!

Number of discs (n)	Number of moves (T)	Pattern for T in terms of n
1	1	
2	3	
3	7	
4		
20		
40		
64		

- How long would it take at 1 move per second for $n = 64$?
- Formula: $T = 2^n - 1$
 $n = 64: T = 2^{64} - 1 = 18,446,744,073,709,551,615$
- Time taken (1 move per sec) = $18,446,744,073,709,551,615 / (60 \times 60 \times 24 \times 365.25 \times 100 \times 10)$
= 584 542 046 Millenia

OWN BRAND: DID HAVE CONCEIT

13 – 3 – 2 – 21 – 1 – 1 – 8 – 5

O, Draconian devil!

Oh, lame saint!

Langdon read the message again and looked up at Fache.

“What the hell does this mean?”

Leonardo of Pisa (Fibonacci) 1170 -1250

Fibonacci series: 1 1 2 3 5 8 13 21 . . .

Rule: $F_{n+2} = F_{n+1} + F_n$ where F_n is the n^{th} term in the series

So: $F_3 = F_2 + F_1 = 1+1=2$; $F_4 = F_3 + F_2 = 2+1=3$; $F_7 = F_6 + F_5 = 13$ etc.

Appearance in Nature http://www.world-mysteries.com/sci_17.htm

Petals on flowers

The number of petals on a flower, that still has all of its petals intact, is for many flowers a Fibonacci number:

3 petals (lily, iris); 5 petals (buttercup, wild rose, larkspur, columbine);

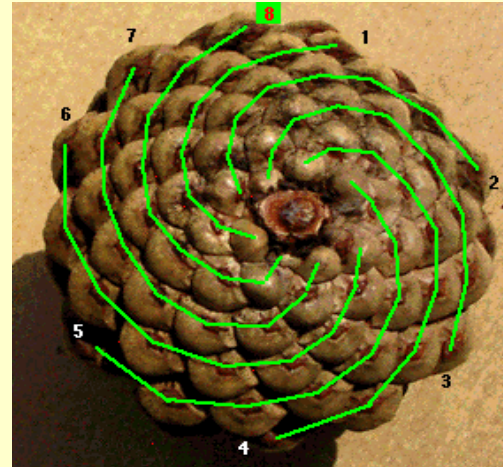
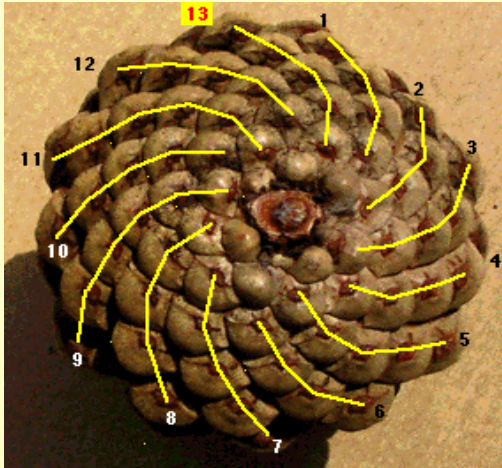
8 petals (delphiniums); 13 petals (ragwort, corn marigold, cineraria);

21 petals (aster, black-eyed susan, chicory); 34 petals (plantain, pyrethrum);

55, 89 petals: michaelmas daisies, the asteraceae family

Some species are very precise about the number of petals they have - e.g. buttercups, but others have petals that are very near those above, with the average being a Fibonacci number. 22

Pine Cone (two sets of spirals)



Golden Rectangle

Find some more members of the Fibonacci series.

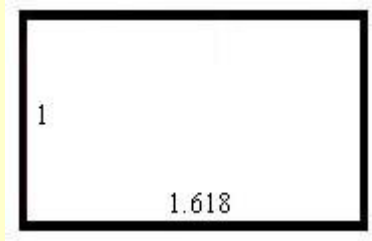
1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597...

F_{n+1}/F_n : 1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21, 55/34, 89/55...

F_{n+1}/F_n : 1, 2, 1.5, 1.67, 1.6, 1.625, 1.61538, 1.61765, 1.61818...

Golden Rectangle

$$F_{n+1}/F_n \rightarrow 1.618033989..$$

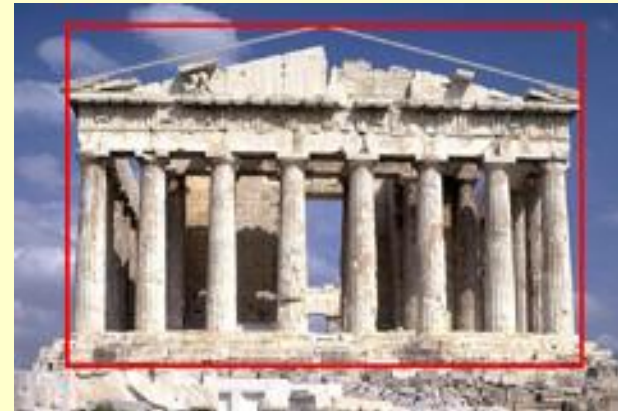


$$\text{Calculate } (1+\sqrt{5})/2 =$$

Appearance in Art and Architecture

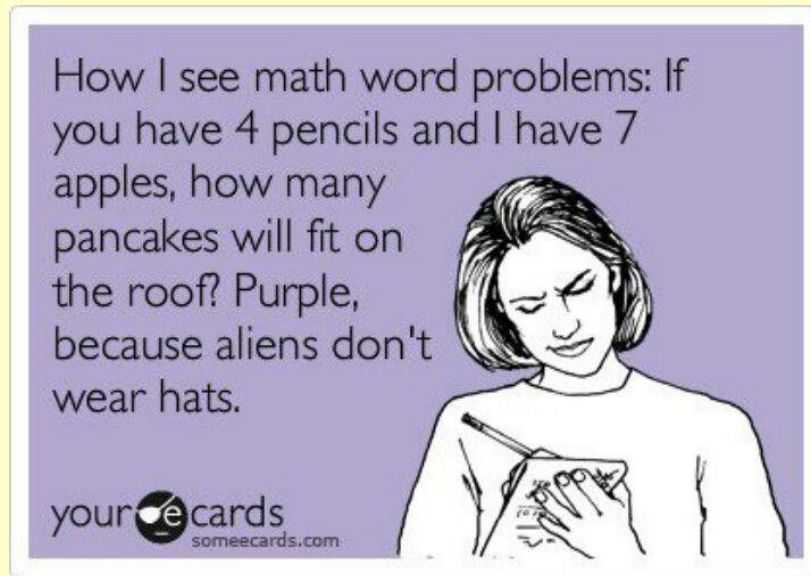


Many appearances in Leonardo da Vinci's work



The Parthenon (restored dimensions)

Word Problems (old as the hills)



Modern textbook: Two bees working together can gather nectar from 100 hollyhock blossoms in 30 minutes. Assuming that each bee works the standard 8 hour day, 5 days a week, how many blossoms do these bees gather nectar from in a summer season of 15 weeks?

Snippets from history



The scribe of the Rhind Mathematical Papyrus (British Museum), an Egyptian document more than 3,600 years old, introduces 84 problems by saying that he is presenting the “**correct method of reckoning, for grasping the meaning of things and knowing everything that is, obscurities and all secrets.**”

The copyist noted that he was writing in year 33 of the reign of Apophis, the penultimate king of the Hyksos Fifteenth Dynasty (about 1650-1550 BC)

An old English riddle

As I was going to St. Ives I met a man with seven wives,
Every wife had seven sacks, every sack had seven cats,
Every cat had seven kits: kits, cats, sacks and wives,
How many were going to St. Ives?

The Rhind Mathematical Papyrus contains a puzzle of sevens with a similar ring.

Seven houses have seven cats that each eat seven mice that each eat seven grains of barley. Each barley grain would have produced seven hekats of grain. (A hekat was a unit of volume, roughly 1.3 gallons.)

The goal: to determine how many things are described.

The answer: 19,607. (The method: $7 + 7^2 + 7^3 + 7^4 + 7^5$.)

Catholic Education (circa 1808)

If a cardinal can pray a soul out of purgatory by himself in an hour, a bishop in three hours, and a priest in five, in what time can they pray out three souls all praying together?

More snippets from history

A mathematical epitaph



Tomb inscription (Diophantus circa 250 AD)

'Here lays Diophantus, the wonder behold. Through art algebraic, the stone tells how old'

'God gave him his boyhood one-sixth of his life;

One twelfth more as youth while whiskers grew rife;

And then yet one-seventh ere marriage begun;

In five years there came a bouncing new son.

Alas, the dear child of master and sage After attaining half the measure of his father's life chill fate took him.

After consoling his fate by the science of numbers for four years, he ended his life.'

His age? (84)

Still more snippets from history

Problems from India and Arabia (~ 850 AD)

- An ape and an eagle lived at the top of a cliff one hundred yards from the base of the cliff. The eagle flew up into the air a certain distance, and then flew straight to the village. The ape scrambled down the cliff and ran to the village. The distance traversed by the two was the same.

What height upwards did the eagle fly? (many answers possible)

- A powerful unvanquished excellent black snake which is 80 angulas in length, enters into a hole at the rate of $7\frac{1}{2}$ angulas in $\frac{5}{14}$ of a day, and in course of a day its tail grows $1\frac{1}{4}$ of an angula.

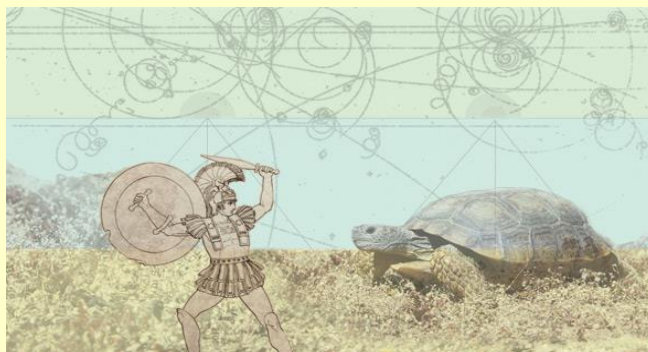
O ornament of arithmeticians, tell me by what time this serpent enters fully into the hole. ($\frac{10}{3}$ days)

- The third part of a necklace of pearls, broken in an amorous struggle, fell to the ground; its fifth part rested on the couch; the sixth part was saved by the wench, and the tenth part was taken by her lover. Six pearls remained strung.

Say of how many pearls was the necklace composed. (60)

Paradoxes

Achilles and the Tortoise



Achilles (the famous Greek athlete) is engaged in a race with a tortoise, which has been granted a start. Achilles' task initially seems easy, but he has a problem - before he can overtake the tortoise, he must first catch it.

Suppose the tortoise is given a 10 metres start and Achilles can run 10 times as fast as the tortoise.

When Achilles reaches the starting point the tortoise is 1 metre ahead

When Achilles reaches this point the tortoise is $1/10$ metres ahead

When Achilles reaches this point the tortoise is $1/100$ metres ahead etc

Achilles can never catch the tortoise!

Russell's paradox

In a certain village the barber shaves all those and only those who do not shave themselves. Who shaves the barber?

Bertrand Russell sent the paradox (expressed in set theoretic terms) in a letter to Gottlob Frege as he was completing *Grundlagen der Arithmetik*. It invalidated much of the rigor of his work and Frege added a note saying, "A scientist can hardly meet with anything more undesirable than to have the foundation give way just as the work is finished. I was put in this position by a letter from Mr. Bertrand Russell when the work was nearly through the press."

Bertrand Russell "As I think about acts of integrity and grace, I realise there is nothing in my knowledge to compare with Frege's dedication to truth. His entire life's work was on the verge of completion, much of it ignored to the benefit of men infinitely less capable, and upon finding that his fundamental assumption was in error, he responded with intellectual pleasure clearly submerging any feelings of personal disappointment. It was a telling indication of man's capability of dedication to creative work and knowledge instead of cruder efforts to dominate and be known."

And a conundrum

Three friends go to a café for a snack and receive a combined bill for \$30.

They each put \$10 on the table, which the waiter collects and takes to the till where the cashier informs him that the bill should only be for \$25, and returns \$5 to the waiter in \$1 coins.

The waiter realises he cannot divide the coins equally between the friends so he decides to put \$2 in his own pocket and give \$1 back to each of the friends.

So each of the friends paid \$9 and the waiter has \$2 in his pocket.

Now $3 \times 9 + 2 = 29$ but the friends originally handed over \$30.

Where is the missing \$1?

Life of π

Bible: Solomon circa 980 BC

And he made a molten sea, ten cubits from one brim to the other: it was round all about, and the height was five cubits, and a **line** of thirty cubits did compass it about. (1 Kings 7:23).

(circumference = 3 x diameter: $C = \pi d = 2\pi r$)

In Hebrew, each letter equals a certain number, and a word's "value" is equal to the sum of its letters. In 1 Kings 7:23, the word "line" is written Kuf Vov Heh, but the Heh does not need to be there, and is not pronounced. With the extra bit, the word has a value of 111, but without it, the value is 106. (Kuf=100, Vov=6, Heh=5). The ratio $\pi/3$ is very close to the ratio of 111 to 106. In other words, $\pi/3 = 111/106$ approximately; solving for π , we find $\pi = 3.1415094...$

<http://www.math.rutgers.edu/~cherlin/History/Papers2000/wilson.html>

Rhind papyrus (Egypt) circa 1650 BC

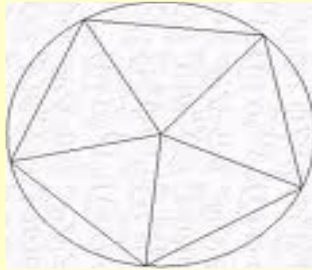
Area of circle = $(8d/9)^2$

Gives $A = (16r/9)^2$ since diameter = 2 x radius

$$= 256r^2/81$$

$$= 3.1605 r^2 \quad (\pi = 3.1605)$$

Ptolemy (150 AD)



Obtained, using chords of a circle and an inscribed 360-gon, the approximation

$$\pi = 3 \frac{17}{120} = 3.14166$$

Euler (18th century) Nothing to do with circles!

$$\pi^2/6 = 1/2^2 + 1/3^2 + 1/4^2 + 1/5^2 + \dots$$

Applications involving Pi

Planetary motion, mechanical engine governors, pendulums, musical notes, statistics, electricity, magnetism, tidal movements...

China: Zu Chongzhi (430 – 500)

Zu used the [method of exhaustion](#) to inscribe a 12,288-gon. Zu's value of π ($355/113 = 3.14159292$) is precise to eight decimal places and accurate to six). For a thousand years thereafter no subsequent mathematician computed a value this precise.

Pi and politics

A Nazi victim: In 1934 the eminent mathematician Edmund Landau (Jewish) published a text book in which he defined π in terms of trigonometry, a highly appropriate mathematical approach. He was subsequently dismissed from the University of Gottingen. Another eminent mathematician, Bieberbach, described the dismissal as follows.

“Thus the valiant rejection which a great mathematician has experienced is due in the final analysis to the fact that the un-German style of this man in his research and teaching is unbearable to German feelings. A people who have perceived how members of another race are working to impose ideas foreign to its own must refuse teachers of an alien culture.”

Indiana legislation: Pi bill No 246 (1897)

Certainly the most bizarre of all efforts to determine the value of pi.

'It has been found that the ratio of the diameter and circumference is as five-fourths to four...'

Note that this means: $C/d = 4/(5/4) = 16/5 = 3.2$

Bill passed its third reading in the House by a vote of 67 to 0. In the Senate it was read once and referred to the committee on Temperance!

It was read a second time and postponed indefinitely – there it remains.

Modern developments (lunacies!)

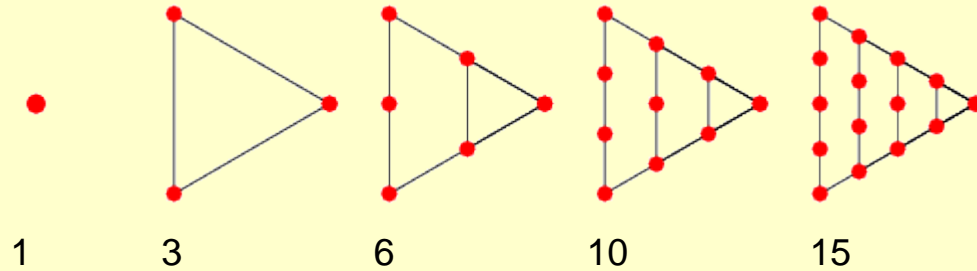
- On November 17, 2016 , after 105 days of round the clock computation, pi enthusiast Peter Trueb's computer finally calculated 22,459,157,718,361 fully verified digits of pi.
- World's most boring book (Guinness): Pi to 100 million decimal places (2007).
- *Reciting Pi*: A Japanese mental health counsellor recited pi to 100,000 decimal places from memory in October 2006, setting what he claims to be a new world record. Akira Haraguchi, 60, needed more than 16 hours to recite the number to 100,000 decimal places, breaking his personal best of 83,431 digits.

- Eccentricity aka March 2010 (see youtube): a young lady Lauren who calls herself [Bookonmyhead](#) balanced 15 books on her head while manipulating a Rubik's cube and reciting the mathematical value of π to a hundred digits!
- International 'pi' day is March 14 (3/14 – as in 9/11).
- *The story of numbers, their minders, and keepers (sane or insane) never ends*

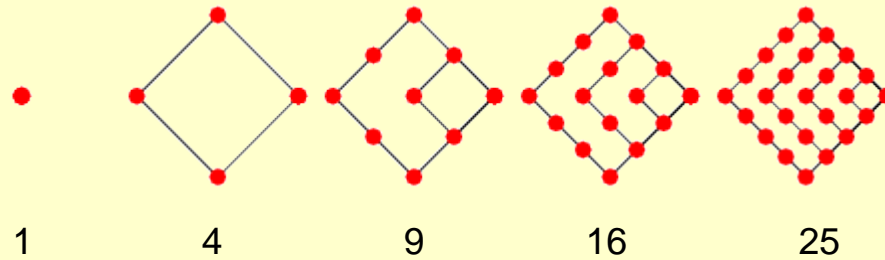


Figurate Numbers

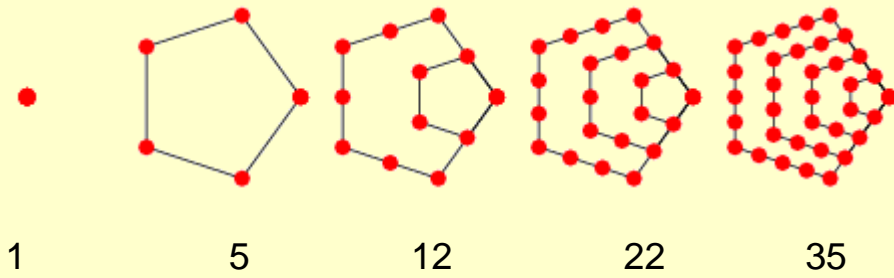
Triangular numbers $[n(n+1)/2]$



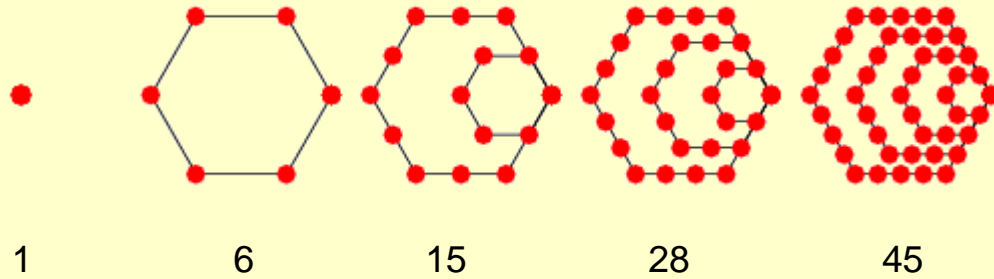
Square numbers $[n^2]$



Pentagonal numbers $[n(3n-1)/2]$



Hexagonal numbers $[n(2n-1)]$



Octagonal numbers $[n(3n-2)]$

